

Quality by Design Scenario Survival/Failure/Reliability Analysis

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Scenario and Technical Approach to Reliability Under Strain

Scenario: Manufacturers face intense global competition, pressure for shorter product-life cycles, stringent cost constraints, and higher customer expectations of quality and reliability. In response to these needs, manufacturing industries have gone through a revolution in the use of statistical methods for product quality. A central element of this approach is a focus on product reliability, typically defined as “quality over time” centered on how long a component can be used until it fails.

Design for reliability requires a core understanding of anticipated and unexpected product failure modes. Important product areas include structural fibers (e.g., polymer and carbon fibers) used in chords (e.g., tire cord) and composites (e.g., carbon fiber composites). Although failure as a function of time (survival: time to failure) is commonly analyzed, failure as a function of some applied strain (survival: tension to failure) is equally appropriate. The only requirement is that this applied strain be measurable and is always a positive quantity.

Design for reliability often involves analyses where a treated or modified product is evaluated in comparison to a control group to address the following questions:

- What level of strain on average can the product tolerate before it fails?
- Does a particular treatment or modification results in higher reliability under strain?
- What are the risk factors that affect reliability under strain?

Technical Approach: There are typically two measurements associated with reliability under strain: the strain value (e.g., tension) and a so-called censoring indicator. Censored data results from partial or censored data—that is, the observation (increasing strain) ends before the event of interest (failure) is observed. Managing censored information is critical

element in survival/failure/reliability analysis. Ignoring censored data or treating them as uncensored measurements can lead to substantially biased result and inaccurate interpretations.

Application of survival/failure/reliability to manufacturing data is critical to Quality-by-Design and Design-for-Reliability. Unfortunately, desktop productivity software like Microsoft Office Word and Excel lack built-in facilities for reliability analysis. To address this shortcoming, manufacturing workers can take one of the following routes. One, purchase, learn and deploy specialized software directed at reliability analysis. This is a costly and inefficient strategy, which ultimately requires multiple import and cut-and-paste operations to produce a results document. And two, acquire custom add-ins for Excel that provide support for reliability analysis. This is both expensive and inflexible as the add-ins cannot be modified or extended to include additional features necessary for analysis.

But there is a better way. Using Inference for .NET in conjunction with Numerical Algorithms Group Numerical C Library (Mark 8) provides a quick, cost effective and reliable means to add survival/failure/reliability capability to desktop productivity software like Microsoft Word and Excel. And you can choose from your preferred, easy-to-learn scripting language like IronPython, IronRuby, managed Jscript or Dynamic VB. The use of IronPython in reliability analysis is illustrated in the following examples.

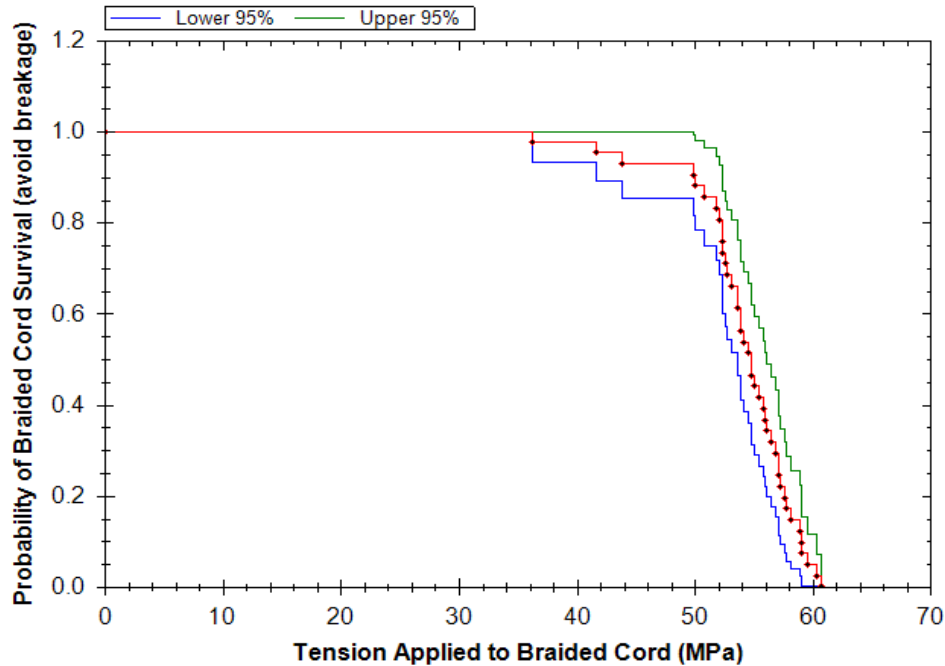
Example 1: Survival of Braided Cord Subject to Strain

Objective of the Study

The objective of the study is to estimate the reliability of a fiber product in terms of the probability that a braided cord survives (does not break) beyond a certain level of applied strain (tension). Specifically, the CordData dataset contains measurements of 48 sections of braided cord, which were placed under increasing tension. During the course of taking the measurements, seven (7) of the braided cords became damaged and the corresponding measurements were stopped. This resulted in censored measurements (Censor=1) where the measurement before stopping the experiment indicated that the breaking tension is at least as large as the final measurement. By contrast, the other measurements (Censor=0) correspond to the tension that resulted in breaking the braided cord.

Analysis of the CordData Dataset and Computation of the Survival Curve

The data set can be analyzed by applying the Kaplan-Meier (Product-Limit) method to Estimate survivor function, which estimates the survival probability for a sample of tension failures. We will use the NAG function `nag_prod_limit_surviv_fn` to compute the Kaplan-Meier estimates of survival probabilities for the CardData tension failures.



Note that the median survival tension is estimated to be 54.8. So if a braided cord is subjected to weathering, it has a 50% probability to surviving—that is, not breaking—to at least an applied tension of 54.8 MPa. The plot shows the survival curve including the 95% confidence intervals based on Greenwood’s formula.

Example 2: Survival of Single Carbon Fibers Subject to Strain

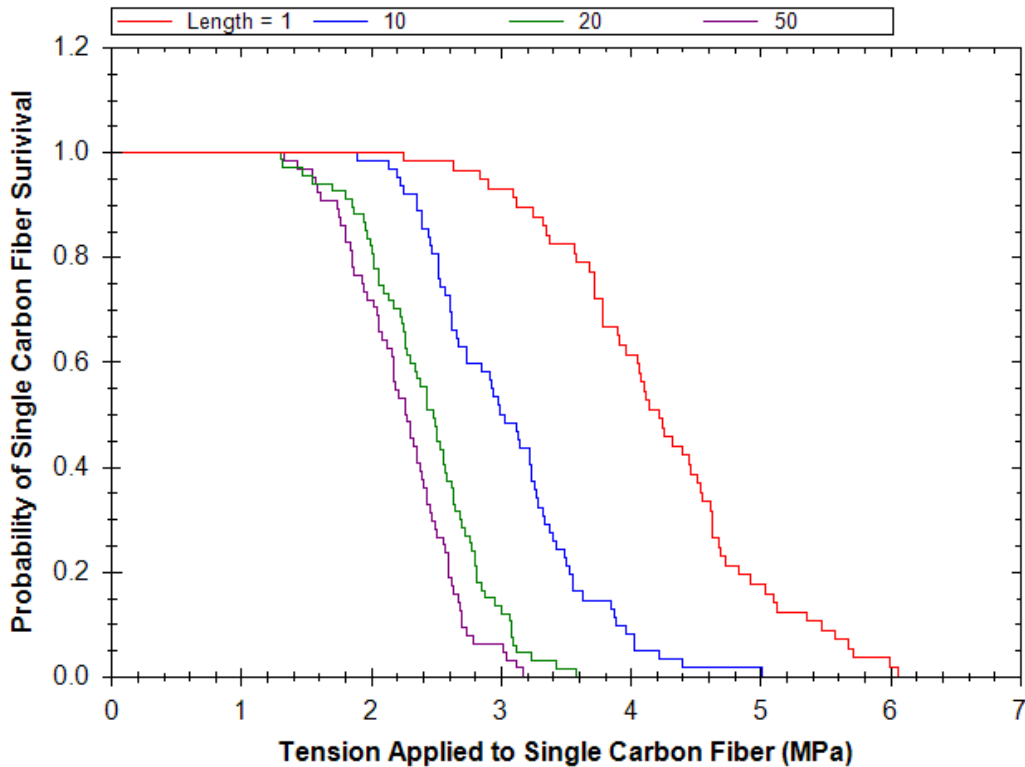
Objective of the Study

High-tech composites, like carbon fiber composites, exploit the qualities of fiber products to achieve strength with the least possible mass. Towards that end, there is a need to understand clearly the factors that affect strength and how to manipulate them to design the material response to strain. Accordingly, the objective of the study was to compare reliability of single carbon fibers subject to tension across four (4) groups corresponding to fibers of different lengths.



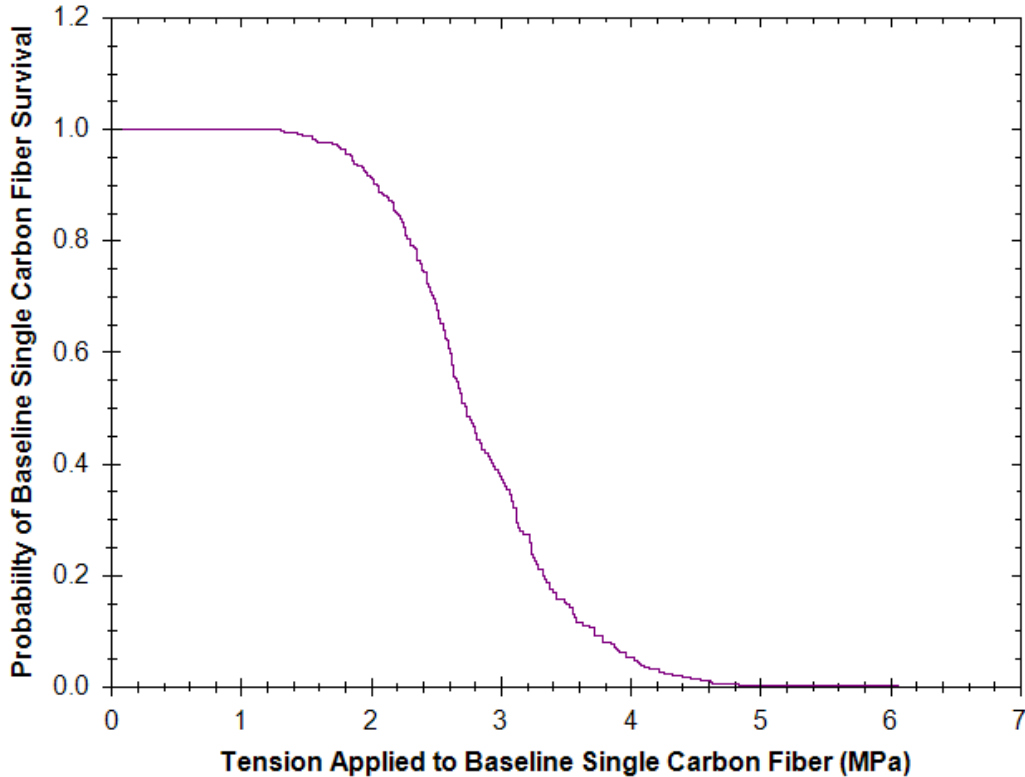
Computation of Survival Curves Across Groups for Carbon Fiber Data

We often need to know whether survival is influenced by one or more factors, called "predictors" or "covariates, such as the length of the fiber". Here we use the Kaplan-Meier method to compute estimates of survival curves for the four different groups corresponding to different fiber lengths.



Cox Proportional Hazard Model for Carbon Fiber Data

Sometimes we need a special kind of regression that lets us assess the effect of each predictor on the shape of the survival curve. The Cox proportional hazard model provides such a regression. To understand the method of proportional hazards, first consider a "baseline" survival curve. This can be thought of as the survival curve of a hypothetical "completely average" carbon fiber—that is, a carbon fiber for which the fiber length is equal to the average value of fiber lengths under study. This is illustrated in the estimation below using the NAG C Library function `nag_surviv_cox_model`.



In the Cox proportional hazard model, the baseline survival curve is systematically "flexed" up or down by the fiber length predictor variable, while still keeping its general shape. The proportional hazards method computes a coefficient for the predictor variable that indicates the direction and degree of flexing that the predictor has on the survival curve. Zero means that a variable has no effect on the curve—that is, it is not a predictor at all; a positive variable indicates that larger values of the variable are associated with greater mortality.

	Covariant Coefficient: β	e^β	Standard Error	Score Function: $U(\beta)$
Length	0.0492	1.05	0.00403	-2.241e-007

Using these coefficients, we can construct a "customized" survival curve for any particular carbon fiber length. More importantly, the method provides a measure of the sampling error associated with the predictor's coefficient. This lets us assess whether the carbon fiber length coefficient is significantly different from zero—that is, whether the carbon fiber length is significantly related to carbon fiber survival when subjected to tension.